# A sharper definition of 'scale-free' provides better insights into networks

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A network can be anything in which people or things are connected to one another: airports between which scheduled flights take place, people who can infect one another with diseases and the routers that connect computers via the internet. A better mathematical view of so-called "scale-free' networks helps to identify, for example, vulnerabilities in the

network. Researchers from Northeastern University in Boston and TU Eindhoven have together drawn up a new mathematical definition of "scale-free." The lack of such a definition had put the value of decades of research in jeopardy. A remarkable dispute in the field of network research needed to be settled.

An important characteristic of networks is the amount of connections that points within the network have. If you want to spread fake news on a social network, the people with a lot of friends are the most interesting when it comes to planting this malicious news. These kinds of network characteristics (like the fact that many people on social media only have a few hundred connections and a handful of them have millions) can be used to make important decisions in a highly targeted manner. This is because such networks have special properties. Many networks, for example, are "scale-free": people do not have a typical number of friends. Although most people have between 100 and 1000, the average (or the median) is not a good description of the network because there are also people with a hundred thousand or even millions of friends. It is precisely these few people with a huge number of friends who do not allow themselves to be captured in an average while still being important to an understanding of the network. In other words: there is no typical "scale" for the number of friends in social networks. An example of a network that is not scale-free is the number of friends in a small class of ten children, in which the children have between three and eight friends each (so none of the children have ten times as many friends as anyone else).

## Dispute over definition

Whether a network is scale-free or not is very interesting for network scientists, as scale-free networks are either very robust or fragile. The exact definition of scale-free is the subject of fierce debate. Earlier research published in the leading journal *Nature Communications* even

stated that scale-free networks were so rare that the field of network science may have been based on incorrect conclusions for decades.

Network scientists at Northeastern University in Boston and TU Eindhoven have now proposed a better mathematical definition. In addition, they have also proposed a method for investigating whether networks are scale-free. The better definition and the method both enable them to better analyze the distribution of the number of connections in networks. As a result, it becomes apparent that it isn't just a few percent of the networks studied that are scale-free, but rather dozens of percent. The new definition makes it possible to more precisely substantiate policies that focus on networks that also fall under the new definition.

## Incorrect picture

This insight potentially saves not only money but also manpower and energy. Remco van der Hofstad, one of the researchers involved, has this to say: "If this scale-free characteristic is not correct, many conclusions will be lost. That has implications for the modeling of the brain and social networks, as well as risk calculations in the economy. Other scientists have made the link, for example, between networks and the economic crisis of 2008. They investigated whether banks were robust, but not at the fact that the entire economy is a network. If you don't include these kinds of network effects, you have an incorrect picture of the potential damage."

The results of the researchers were previously published on arXiv and have now been published in the new open access journal *Physical Review Research*.

**More information:** Ivan Voitalov et al. Scale-free networks well done, *Physical Review Research* (2019). DOI:

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