Theory and experiments to understand a contact between two filaments

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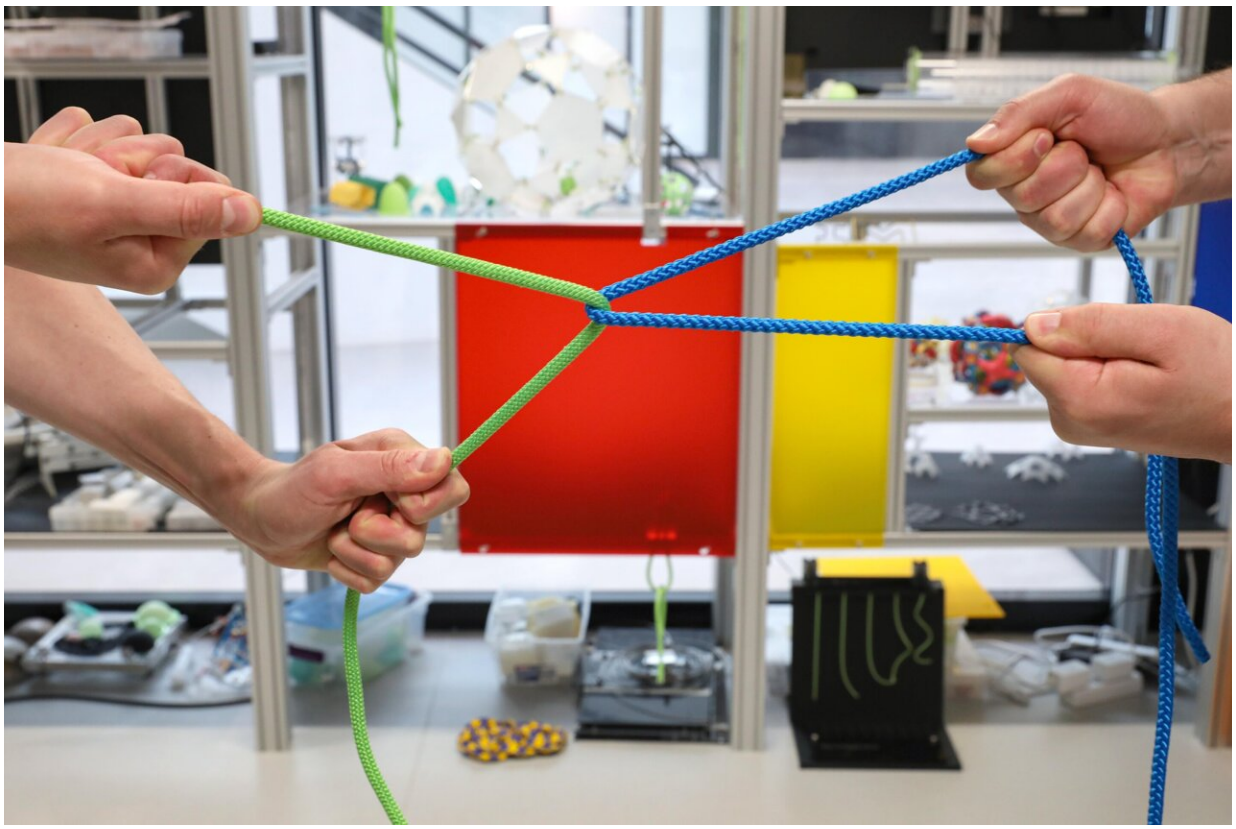

The orthogonal clasp. Credit: 2021 EPFL

Mechanical engineers and mathematicians at EPFL have joined forces to gain a better understanding of the geometry and mechanics of two filaments in contact—as in the cases of knots and woven fabrics.

Pedro Reis, head of EPFL's Flexible Structures Laboratory, and John Maddocks, head of EPFL's Laboratory for Computation and Visualization in Mathematics and Mechanics, have something in common: a fascination with ropes and knots. Reis, an engineer, is an avid rock climber while Maddocks, a mathematician, has a passion for sailing. But their mutual interest in knots is not restricted to their hobbies, as knots are used in a variety of applications—take surgical sutures, for example. And although knots have been part of our daily lives since the dawn of time, their mechanics are still poorly understood.

A simplified knot

Reis, Maddocks and the researchers at their labs have been studying a specific configuration of contact between two filaments—the orthogonal clasp—which can be regarded as the most basic building block for every knot. "This intertwining is the simplest of all knots; or to be more specific, it's the link that knots are based on. It's also the most widely used knot. It's found in the thread patterns in our clothing, for example," says Reis. He, Maddocks and their research team conducted a detailed study of the contact region between the two filaments, and their findings have just been published in *Proceedings of the National Academy of Sciences* (*PNAS*).

For over 30 years, Maddocks has been investigating (among other things) the mathematical theories that explain the mechanics of knots, and in particular the complex geometry of the curves that make up the contact region between filaments. In 2003, his colleague at the time Eugene Starostin published a paper specifically on the orthogonal clasp. The contact zone resembles a diamond shape, and the four corners mark the main pressure peaks. However, his theory could never be empirically corroborated due to technical limitations. "When Pedro and I decided to work together, we wondered whether Starostin's earlier results would still be relevant in practice," says Maddocks. "We then carried out tests, measurements and experiments to answer this question." Reis adds: "the contact region has always been calculated according to an ideal hypothesis, but never experimentally verified."

Checking the initial findings

The researchers at Reis' lab conducted experiments using a tomograph, which employs X-rays and computer models to generate 3D images of objects. "Tomography lets us look inside the contact region between the two filaments. We then corroborated our experimental results with computer simulations. We didn't expect to find such a heterogeneous pressure distribution between the two filaments," says Paul Grandgeorge, a postdoc at Reis' lab. Their experiments showed that the pressure region between two filaments coincided with Starostin's earlier geometrical calculations. "This is a small step forward in understanding filaments in contact," says Maddocks.

The capstan equation

Spurred on by these results, the research team wanted to take things a step further. They therefore studied the contact region between two filaments under the effect of friction. Their initial hypothesis was that friction could be explained by the capstan equation. "The concept behind the capstan equation is simple: when a rope is wrapped around a cylindrical tube, such as a mooring bollard, the tensions in the two hanging strands are separated. The more loops there are around the tube, the greater the difference in tension between the two strands. We assumed that we could use this equation to calculate the tension ratio between the two strands in our experiments," says Grandgeorge.

However, after running several experiments, the researchers concluded that the capstan equation was not applicable in the case of filaments in a frictional state. "The capstan equation assumes that the tube does not deform and is larger in diameter than the rope wrapped around it. In our experiments, however, the elastic rod that functions as the tube can be deformed and has the same diameter as the second rod that functions as the rope," says Reis. But the research team doesn't see these findings as a setback—in fact, quite the opposite. "This gives us even more incentive to find a theoretical model that can explain this physical phenomenon," says Grandgeorge. "It may appear to be a simple problem, but geometrically it's actually quite complicated," adds Maddocks. The researchers expect that there will be many studies conducted on knots in the coming years, bringing a better theoretical understanding to real-world situations.

More information: Paul Grandgeorge et al. Mechanics of two filaments in tight orthogonal contact, *Proceedings of the National Academy of Sciences* (2021). DOI: 10.1073/pnas.2021684118

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