A primal-dual approximation algorithm for the k-prize-collecting minimum vertex cover problem with submodular penalties

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Algorithm 1 The two-phase primal-dual algorithm

Input: An instance $\mathcal{I} = (G; c, \pi; k)$ of the k-PCVCS.
Output: A feasible pair (S, R).

Set $V^{tight} = \emptyset$, $E^{act} = E$, $y_e = 0$ for any $e \in E$ and $\gamma = 0$.
while $E^{act} \neq \emptyset$ **do**
  Keep $\gamma = 0$ and increase $\{y_e\}_{e \in E^{act}}$ simultaneously until either some vertex v becomes tight or some edge set E' becomes tight.
  if *vertex v become tight* **then**
    $V^{tight} := V^{tight} \cup \{v\}$, $E^{act} := E^{act} \setminus \delta(\{v\})$.
  else
    $E^{act} := E^{act} \setminus E'$.
while $|\delta(V^{tight})| < k$ **do**
  Increase $\{y_e\}_{e \in E \setminus \delta(V^{tight})}$ and γ simultaneously until some vertex v becomes tight.
  $V^{tight} := V^{tight} \cup \{v\}$.

A combinatorial 3-approximation algorithm (Algorithm 2) based on the guessing technique and the primal-dual framework. Credit: Liu, X., Li, W. & Yang, J.

The k-prize-collecting minimum vertex cover problem with submodular penalties (k-PCVCS) is a generalization of the minimum vertex cover problem, which is one of the most important and fundamental problems in graph theory and combinatorial optimization.

This problem is to select a vertex set that covers at least k edges, and the objective is to minimize the total cost of the vertices in the selected set plus the penalty of the uncovered edge set, where the penalty is determined by a submodular function.

To solve the k-PCVCS, Xiaofei Liu et al. published their new research in *Frontiers of Computer Science*.

In the research, they first proved that Algorithm 1 can be implemented in $O(n^{16}r+n^{17})$, where r is the time for one function evaluation. Then, they proved that Algorithm 2 is a 3-approximation algorithm for the k-PCVCS. Specifically, if the penalty function is linear, Algorithm 2 is a 2-approximation algorithm.

Future work may focus on studying the version with general penalties, such as, subadditive or supermodular penalties. Meanwhile, the k-PCVCS with hard capacities deserves to be explored, in which each vertex v is covered at most C_v edges.

More information: Xiaofei Liu et al, A primal-dual approximation algorithm for the k-prize-collecting minimum vertex cover problem with submodular penalties, *Frontiers of Computer Science* (2022). DOI: 10.1007/s11704-022-1665-9

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